A Two counter machine in CCS

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A *two counter machine* is a machine composed by two unlimited counters that can be incremented, decremented and checked for zero, and a Finite State Automaton (actually, a Transducer, since it can have outputs) that controls the operations over the two counters, defining the function computed by the machine. A two counter machine is known to be Turing equivalent, that is, any Turing machine can be expressed as a two counter machine.

A two counter machine can be expressed in CCS as follows:

$C = inc.(C[a\backslash b] \mid b.C) \ /\{b\} + dec.\overline{a}.nil$

$Z = inc.(C[a\backslash b] \mid b.Z) \ /\{b\} + tst.Z$

$Counter = Z$

$Counter1 = Counter[inc\backslash inc1, dec\backslash dec1, tst\backslash tst1]$

$Counter2 = Counter[inc\backslash inc2, dec\backslash dec2, tst\backslash tst2]$

$TwoCounterMachine = (Counter1 \mid ASF \mid Counter2) \ /\{inc1, inc2, dec1, dec2, tst1, tst2\}$

where ASF is the Finite State Automaton that defines the function of the two counter machine.

This implies that CCS is Turing equivalent. Notice that all the CCS operators are needed to define a two counter machine.